

The Master Theorem

When presented with a recurrence of the form:

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

We can find an asymptotically tight bound by examining the following three cases:

Case 1

$$f(n) \in O\left(n^{\log_b a - \varepsilon}\right) \text{ for a } \varepsilon > 0$$

$$T(n) \in \Theta\left(n^{\log_b a}\right)$$

Case 2

$$f(n) \in \Theta\left(n^{\log_b a}\right)$$

$$T(n) \in \Theta\left(n^{\log_b a} \cdot \log(n)\right)$$

Case 3

$$f(n) \in \Omega\left(n^{\log_b a + \varepsilon}\right) \text{ for a } \varepsilon > 0$$

$$T(n) \in \Theta(f(n))$$